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INTERACTION OF SHOCK WAVES WITH A COMBINED DISCONTINUITY IN TWO-PHASE MEDIA. 1. EQUILIBRIUM APPROXIMATION

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The problem of interaction of shock waves of various types (completely dispersed, frozen-dispersed, dispersed-frozen, and frozen shock waves with a two-front configuration) with a motionless combined discontinuity in a mixture of two condensed materials is considered. The mathematical description is based on the equations of mechanics of heterogeneous media in a one-dimensional isothermal approximation with allowance for differences in velocities and pressures of the components. In the equilibrium approximation in terms of velocities and pressures of the components, the wave configuration and flow parameters are determined in all equilibrium states behind the incident, transient, and reflected shock waves.

The problem of interaction of shock waves (SW) with a combined discontinuity (CD) separating the mixture with different volume concentrations of the components is of great theoretical and practical interest. Fedorov [1] proposed a mathematical model for the description of the behavior of a mixture of two condensed materials in the two-velocity, two-temperature approximation of mechanics of heterogeneous media with different pressures of the components. The existence of two types of shock waves was established within the framework of one-velocity continua: dispersed SW (with a monotonic velocity profile) and frozen SW (with a discontinuous velocity profile). For a two-component mixture in the two-velocity approximation of mechanics of heterogeneous media with different pressures of the components, it was shown [2, 3] that there can exist four types of shock waves: dispersed, frozen-dispersed, dispersed-frozen, and frozen shock waves with a two-wave configuration. Stability of shock waves of different types to finite and infinitesimal perturbations was studied in [4]. The mechanism of interaction of shock waves of all types with a rigid boundary was studied analytically and numerically in [5, 6]. As a result, the equilibrium parameters of the mixture established after the reflection of the incident SW from the rigid wall were determined, and the possibility of a change in the SW type upon reflection was demonstrated.

In the present work, we study the process of interaction of the incident SW and CD for the case where the relaxation times of velocities and pressures of the components of the mixture are close to zero (equilibrium flow) and also for the general case of finite relaxation times.

Physicomathematical Formulation of the Problem. We consider a mixture of a liquid and solid particles with an SW propagating from right to left with a velocity D. At a certain time, the SW reaches the CD separating regions with different volume concentrations of particles and interacts with it. We have to determine the flow pattern after interaction of the incident SW and CD.

For the mathematical description of this process in the one-dimensional isothermal approximation of mechanics of a heterogeneous mixture of condensed media with different pressures and velocities, we use the laws of conservation of mass and momentum for each component of the mixture, which are supplemented by the equation of m_2 transfer and equations of state:

$$
\begin{gathered}
\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1 u_1}{\partial x} = 0, \quad \frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2 u_2}{\partial x} = 0, \quad \frac{\partial \rho_1 u_1}{\partial t} + \frac{\partial \rho_1 u_1^2}{\partial x} = -m_1 \frac{\partial P_1}{\partial x} + F_{\rm St}, \\
\frac{\partial \rho_2 u_2}{\partial t} + \frac{\partial \rho_2 u_2^2}{\partial x} = -m_2 \frac{\partial P_2}{\partial x} - (P_2 - P_1)\frac{\partial m_2}{\partial x} - F_{\rm St}, \quad \frac{\partial m_2}{\partial t} + u_2 \frac{\partial m_2}{\partial x} = R, \\
m_1 = 1 - m_2, \qquad P_1 = \rho_1/m_1 - 1, \qquad P_2 = a^2(\rho_2/m_2 - \bar\rho).
\end{gathered} \tag{1}
$$

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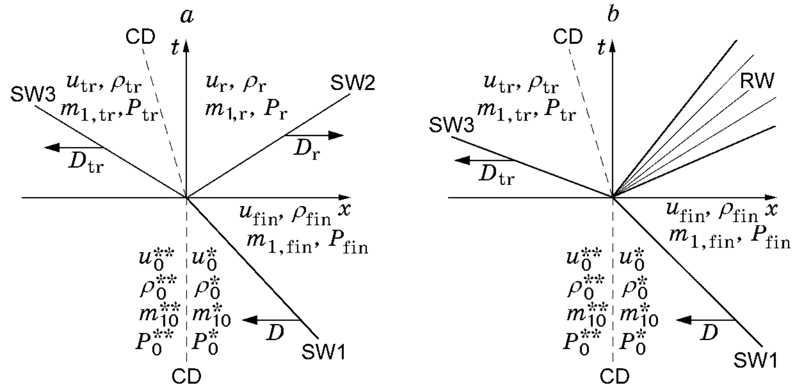

Fig. 1. Interaction of the incident SW1 and CD: reflection of SW2 (a) and reflection of RW (b).

Here $\rho_i = m_i\rho_{ii}$, u_i, P_i, and m_i are the mean density, velocity, pressure, and volume concentration of the ith component of the mixture, $F_{\rm St} = m_1\rho_2(u_2 - u_1)/\tau_{\rm St}$ is the Stokes force, $\tau_{\rm St} = 2\bar{\rho}/(9\mu_1)$ is the time of the Stokes relaxation of velocities, $R = m_1m_2(P_2 - P_1)/\tau_{m_2}$ is the function that describes the transfer process of the solid phase, $\tau_{m_2} = 2\rho_{22,0}a_2r/(\rho_{11,0}a_1^2) \approx 2\mu_2$ is the time of pressure relaxation of the components of the mixture, μ_i is the dynamic viscosity of the ith component, ρ_{ii} is the true density of the ith component, and a_i and $\rho_{ii,0}$ are the velocity of sound and true density of the material of the ith component of the mixture. The subscripts 1 and 2 refer to the parameters of the light component (carrier phase) and heavy component (discrete phase) of the mixture, respectively.

In system (1), the velocities were normalized to a_1, the densities to $\rho_{11,0}$, the pressure to $a_1^2\rho_{11,0}$, the time t to $t_0 = r/a_1$, μ_i to $a_1\rho_{11,0}r$, and the spatial variable x to the radius of solid particles r. Thus, at $t = 0$, the dimensionless velocity of sound and the true density of the light component are equal to unity, and these parameters for the heavy component are $a = a_2/a_1$ and $\bar{\rho} = \rho_{22,0}/\rho_{11,0}$, respectively.

The initial data for system (1) can be represented in the form

$$t = 0: \qquad \boldsymbol{\varphi} = \boldsymbol{\varphi}^* \text{ for } x \geqslant x_0, \quad \boldsymbol{\varphi} = \boldsymbol{\varphi}^{**} \text{ for } x < x_0, \tag{2}$$

where the vector of flow parameters in the initial equilibrium state is denoted as $\boldsymbol{\varphi}^*$ on the right of the CD (it describes the flow in the form of a steady SW of some type [3]) and as $\boldsymbol{\varphi}^{**}$ on the left of the CD; x_0 is the coordinate of the CD boundary. We have to find a solution of system (1), (2) for $t \geqslant 0$.

Calculation of Flow Parameters in the Equilibrium Approximation. Interaction of the incident SW and CD in the equilibrium approximation is described by the Riemann problem in a mixture of two compressible gases. The wave pattern of interaction of the incident SW and CD is shown in Fig. 1.

Figure 1a corresponds to the case, where the incident SW1 (propagating with a velocity D) decomposes, after interaction with the CD, into the reflected SW2 ($D_{\rm r}$) and transient (refracted) SW3 ($D_{\rm tr}$). The transient SW3 moves in the same direction as the incident SW1 with respect to the CD, and the reflected SW2 moves in the opposite direction. Prior to interaction with the incident SW1, the quiescent CD is located at the point $x_0 = 0$. The mixture ahead of the front of the incident SW1 is in the equilibrium state; its parameters are denoted by the subscript 0. In the region between the front of the incident SW1 and motionless boundary of the CD, the mixture is characterized by equilibrium initial parameters marked by the superscript "$*$"; the parameters of the mixture behind the CD are denoted by the superscript "$**$." In this case, we assume that $m_{10}^{**} < m_{10}^*$, i.e., $\rho_0^{**} > \rho_0^*$ [since $\rho_0 = \bar{\rho} + m_{10}(1 - \bar{\rho})$]. An equilibrium state is formed behind SW1; its parameters are marked by the subscript "fin." After interaction of the incident SW1 and CD, the latter starts to move behind the transient SW3 with a velocity $u_{\rm CD}$. Its trajectory is shown by the dashed curve in Fig. 1a. The reflected SW2 propagates over the mixture with parameters marked by the subscript "fin"; new equilibrium parameters marked by the subscript and superscript "r" are established behind its front. The transient SW moves over the mixture with parameters denoted by the superscript "$**$"; equilibrium parameters marked by the subscript "tr" are established behind its front.

Thus, based on the wave pattern proposed, we have to determine the velocity of the transient SW3, reflected SW2 and CD, and also other flow parameters established after interaction of the incident SW1 and CD for given initial parameters (velocity of the incident SW1 and initial volume concentrations ahead of the CD and behind it).

Figure 1b corresponds to the case, where the incident SW1 decomposes, after interaction with the CD, into the transient SW3 and a fan of rarefaction waves (RW) reflected from the CD boundary and moving in the opposite direction. As is shown in [7], the fore front of the RW propagates with a velocity varying, generally speaking, from equilibrium-frozen ($C_{\text{e.f.}}$) at the initial time to equilibrium (C_{e}) at large times; the rear front of the RW propagates with a constant velocity corresponding to the velocity of sound of the light component. Here $m_{10}^{**} > m_{10}^{*}$ and $\rho_0^{**} < \rho_0^{*}$.

We study the case shown in Fig. 1a, where the incident SW1, interacting with the CD, decomposes into two shock waves (reflected and transient).

To solve this problem, we use the laws of conservation of mass and momentum on the incident SW1, reflected SW2, and refracted SW3, the conditions on the CD before and after its interaction with the incident SW1, and equations of state written for the corresponding equilibrium states.

The following conditions are imposed on the CD:

1) in the initial equilibrium state (before interaction with SW1), $u_0^* = u_0^{**} = u_0 = 0$ and $P_0^* = P_0^{**} = P_0 = 0$;

2) in the final equilibrium state (after interaction with SW1), $u_{\text{tr}} = u_{\text{r}} = u_{\text{CD}}$ and $P_{\text{tr}} = P_{\text{r}} = P_{\text{CD}}$.

The following conditions are imposed on the SW:

1) on the incident SW1,

$$\rho_0^*(u_0^* - D) = \rho_{\text{fin}}(u_{\text{fin}} - D), \qquad P_0^* + \rho_0^*(u_0^* - D)^2 = P_{\text{fin}} + \rho_{\text{fin}}(u_{\text{fin}} - D)^2;$$

2) on the reflected SW2,

$$\rho_{\text{fin}}(u_{\text{fin}} - D_{\text{r}}) = \rho_{\text{r}}(u_{\text{r}} - D_{\text{r}}), \qquad P_{\text{fin}} + \rho_{\text{fin}}(u_{\text{fin}} - D_{\text{r}})^2 = P_{\text{r}} + \rho_{\text{r}}(u_{\text{r}} - D_{\text{r}})^2;$$

3) on the refracted (transient) SW3,

$$\rho_0^{**}(u_0^{**} - D_{\text{tr}}) = \rho_{\text{tr}}(u_{\text{tr}} - D_{\text{tr}}), \qquad P_0^{**} + \rho_0^{**}(u_0^{**} - D_{\text{tr}})^2 = P_{\text{tr}} + \rho_{\text{tr}}(u_{\text{tr}} - D_{\text{tr}})^2.$$

Here $P^* = (C_{\text{e.f.}}^*)^2\rho + m_2^*C - 1$, $P^{**} = (C_{\text{e.f.}}^{**})^2\rho + m_2^{**}C - 1$, $(C_{\text{e.f.}}^*)^2 = \xi_1^* + a^2\xi_2^*$, $m_2^* = 1 - m_1^*$, $m_2^{**} = 1 - m_1^{**}$, $m_1^* = (C + \rho(C_{\text{e.f.}}^*)^2 - \sqrt{(C + \rho(C_{\text{e.f.}}^*)^2)^2 - 4C\rho\xi_1^*})/(2C)$, $(C_{\text{e.f.}}^{**})^2 = \xi_1^{**} + a^2\xi_2^{**}$, and $m_1^{**} = (C + \rho(C_{\text{e.f.}}^{**})^2 - \sqrt{(C + \rho(C_{\text{e.f.}}^{**})^2)^2 - 4C\rho\xi_1^{**}})/(2C)$.

We introduce the following notation: $u_0^* - D = U_0^*$, $u_0^{**} - D_{\text{tr}} = U_0^{**}$, $u_{\text{r}} - D_{\text{r}} = U_r$, $u_{\text{fin}} - D = U_{\text{fin}}$, $u_{\text{tr}} - D_{\text{tr}} = U_{\text{tr}}$, and $u_{\text{fin}} - D_{\text{r}} = U_{\text{fin}}^{\text{r}}$.

Since SW1 moves with a velocity D from right to left over the mixture with the parameters ρ_0^*, U_0^*, m_{10}^*, and P_0^*, the mixture behind the wave front acquires a new equilibrium state with the parameters ρ_{fin}, U_{fin}, $m_{1,\text{fin}}$, and P_{fin}, which are found from the conditions on the incident SW1. We have $\rho_{\text{fin}} = \rho_0^*U_0^*/U_{\text{fin}}$ from the law of conservation of mass and $P_{\text{fin}} = \rho_0^*U_0^*(U_0^* - U_{\text{fin}})$ from the law of conservation of momentum. Using the equation of state $P_{\text{fin}} = (C_{\text{e.f.}}^*)^2\rho_{\text{fin}} + m_{2,\text{fin}}^*C - 1$ and the expression for the volume concentration of the heavy component obtained from the condition of equal pressures of the components in the final equilibrium state behind the incident SW ($P_{1,\text{fin}} = P_{2,\text{fin}}$) in the form

$$m_{2,\text{fin}}^* = [C - \rho_{\text{fin}}(C_{\text{e.f.}}^*)^2 + \sqrt{(C + \rho_{\text{fin}}(C_{\text{e.f.}}^*)^2)^2 - 4C\rho_{\text{fin}}\xi_1^*}]/(2C),$$

we find a cubic equation for determining U_{fin}:

$$(\rho_0^*U_0^*)^2U_{\text{fin}}^3 - \rho_0^*U_0^*U_{\text{fin}}^2(2 - C + 2\rho_0^*(U_0^*)^2)$$
$$+ U_{\text{fin}}[1 - C + \rho_0^*(U_0^*)^2(2 - C) + (\rho_0^*U_0^*C_{\text{e.f.}}^*)^2 + (\rho_0^*)^2(U_0^*)^4]$$
$$+ \rho_0^*U_0^*(C\xi_1^* - (C_{\text{e.f.}}^*)^2 - \rho_0^*(U_0^*)^2(C_{\text{e.f.}}^*)^2) = 0.$$

This equation decomposes into two equations:

$$U_{\text{fin}} - U_0^* = 0, \qquad (\rho_0^*U_0^*)^2U_{\text{fin}}^2 - \rho_0^*U_0^*U_{\text{fin}}(2 - C + \rho_0^*(U_0^*)^2) + 1 - C + (\rho_0^*C_{\text{e.f.}}^*U_0^*)^2 = 0.$$

Thus, the solutions have the following form:

$$U_{\text{fin}} = U_0^*, \quad U_{\text{fin}}^{\pm} = \frac{2 - C + \rho_0^*(U_0^*)^2 \pm \sqrt{(C - \rho_0^*(U_0^*)^2)^2 + 4\rho_0^*(U_0^*)^2(1 - \rho_0^*(C_{\text{e.f.}}^*)^2)}}{2\rho_0^*U_0^*}.$$

Based on the physical conditions of the problem, hereinafter we use the value $U_{\text{fin}} = U_{\text{fin}}^-$. Based on the found value of U_{fin}, we determine ρ_{fin} and P_{fin}, and, then, the values of $m_{2,\text{fin}}$ and $m_{1,\text{fin}}$. In what follows, the values of ρ_{fin}, U_{fin}, $m_{1,\text{fin}}$, and P_{fin} are assumed to be known.

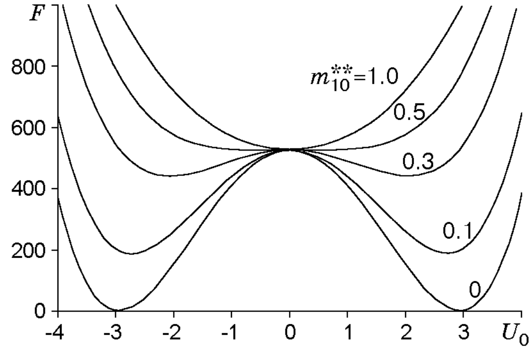

Fig. 2. Biquadratic function $F(U_0)$.

From the conditions on the reflected SW2, we find the dependences $\rho_{\rm r}$ and $P_{\rm r}$ on $U_{\rm r}$ and $D_{\rm r}$; based on the conditions on the transient SW3, we express $\rho_{\rm tr}$ and $P_{\rm tr}$ via $U_{\rm tr}$ and $D_{\rm tr}$. Finally, we obtain the following system of equations:

$$P_{\rm r} = P_{\rm fin} + \rho_{\rm fin} U^{\rm r}_{\rm fin}(U^{\rm r}_{\rm fin} - U_{\rm r}), \qquad P_{\rm tr} = \rho_0^{**} U_0^{**}(U_0^{**} - U_{\rm tr}),$$
$$U_{\rm r} + D_{\rm r} = U_{\rm tr} + D_{\rm tr} = u_{\rm CD}, \qquad P_{\rm r} = P_{\rm tr} = P_{\rm CD}. \tag{3}$$

This system is closed by equations of state.

We consider the second equation of system (3). Substituting $P_{\rm tr}$ from the equation of state into the second equation of (3), we obtain a cubic equation for determining $U_{\rm tr}$

$$(\rho_0^{**} U_0^{**})^2 U_{\rm tr}^3 - \rho_0^{**} U_0^{**} U_{\rm tr}^2 (2 - C + 2\rho_0^{**}(U_0^{**})^2)$$
$$+U_{\rm tr}[1 - C + \rho_0^{**}(U_0^{**})^2(2 - C) + (\rho_0^{**} U_0^{**} C^{**}_{\rm e.f.})^2 + (\rho_0^{**})^2 (U_0^{**})^4] + \rho_0^{**} U_0^{**}(C\xi_1^{**} - (C^{**}_{\rm e.f.})^2 - \rho_0^{**}(U_0^{**})^2 (C^{**}_{\rm e.f.})^2) = 0.$$

If we substitute U_0^* for U_0^{**}, ρ_0^* for ρ_0^{**}, and the subscript "fin" for "tr," the polynomial in the left side of the last equation will be similar to the polynomial in the left side of the equation for determining $U_{\rm fin}$. Hence, the equation under study also has three roots: $U_{\rm tr} = U_0^{**}$ (which corresponds to the initial data of the problem) and

$$U^{\pm}_{\rm tr} = \frac{2 - C + \rho_0^{**}(U_0^{**})^2 \pm \sqrt{(C - \rho_0^{**}(U_0^{**})^2)^2 + 4\rho_0^{**}(U_0^{**})^2(1 - \rho_0^{**}(C^{**}_{\rm e.f.})^2)}}{2\rho_0^{**} U_0^{**}}. \tag{4}$$

In what follows, we consider the value $U_{\rm tr} = U^-_{\rm tr}$ (by analogy with $U^-_{\rm fin}$).

We study the radicand of equality (4), which may be represented in the form of the biquadratic equation

$$F(U_0^{**}) = (\rho_0^{**})^2 (U_0^{**})^4 - 2\rho_0^{**}(U_0^{**})^2 (C - 2 + 2\rho_0^{**}(C^{**}_{\rm e.f.})^2) + C^2 = 0,$$

which has the solution

$$U_0^{**} = \pm\sqrt{\Big[C - 2 + 2\rho_0^{**}(C^{**}_{\rm e.f.})^2 \pm \sqrt{(C - 2 + 2\rho_0^{**}(C^{**}_{\rm e.f.})^2)^2 - C^2}\,\Big]/\rho_0^{**}}.$$

For $m_{10}^{**} = 0$, the internal radicand vanishes, and multiple real roots are obtained: $U_0^{**} = \pm\sqrt{-C/\bar\rho}$. For $m_{10}^{**} = 1$, the internal radicand also vanishes, but we obtain two multiple imaginary roots $U_0^{**} = \pm i\sqrt{|C|}$, since $C < 0$. For other values of $m_{10}^{**} \in (0, 1)$, the internal radicand becomes negative, since $\rho_0^{**}(C^{**}_{\rm e.f.})^2 = m_{10}^{**} + a^2\bar\rho m_{20}^{**} < a^2\bar\rho$. Hence, there are no real solutions.

Figure 2 shows the biquadratic function $F(U_0)$ for a mixture of water and quartz sand for different initial volume concentrations of the mixture. As is shown in Fig. 2, there are three extreme points, which, for $m_{10}^{**} \in [0; 0,5)$ correspond to two local minima for $U_{\rm I,II} = \pm\sqrt{(C - 2 + 2\rho_0^{**}(C^{**}_{\rm e.f.})^2)/\rho_0^{**}}$ and a maximum for $U_{\rm III} = 0$. For $m_{10}^{**} = 0.5$, the points of local minima and maximum converge into one point [twice degenerate point of the curve $F(U_0)$] $U_{\rm I} = U_{\rm II} = U_{\rm III} = 0$, where a minimum is reached; for $m_{10}^{**} \in (0.5; 1]$, there exists one real minimum for $U = 0$. It should be noted that the minimum points $U_{\rm I}$ and $U_{\rm II}$ are shifted from $\pm\sqrt{-C/\bar\rho}$ to zero; the value of $F(U_0)$ changes from zero to C^2 for $m_{10}^{**} \in [0; 0.5]$, and the position of the point $U = 0$ is constant, whereas the value of $F(U_0) = C^2$ at this point depends only on the components that constitute the mixture examined. For example, we have $C^2 = 522.1225$ for a mixture of water and quartz sand.

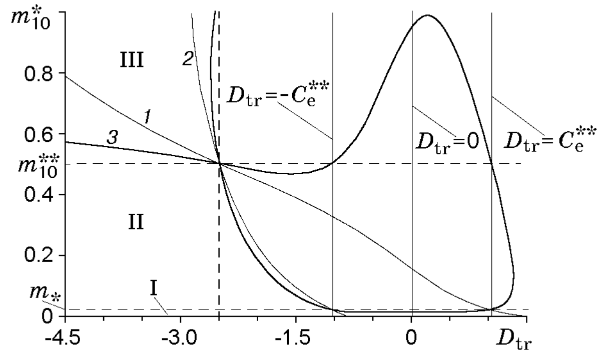

Fig. 3. Roots of Eq. (7) for $D = -2.5$ and $m^{**}_{10} = 0.5$.

Thus, Eq. (4) allows us to determine the relative velocity of the mixture behind the transient SW3 as a function $U_{\rm tr} = U_{\rm tr}(D_{\rm tr})$.

We derive an equation for $D_{\rm tr}$. From the third equation of system (3), which expresses the equality of mass velocities on the CD, we find the relative velocity of the mixture behind the reflected SW2:

$$U_{\rm r} = U_{\rm tr} + D_{\rm tr} - D_{\rm r}. \tag{5}$$

Using the first and second equations of system (3) and the condition of equal pressures on the CD, we obtain the expression

$$P_{\rm fin} + \rho_{\rm fin} U^{\rm r}_{\rm fin}(U^{\rm r}_{\rm fin} - U_{\rm r}) = \rho^{**}_0 U^{**}_0 (U^{**}_0 - U_{\rm tr}),$$

where $U^{\rm r}_{\rm fin} = u_{\rm fin} - D_{\rm r} = U_{\rm fin} - D_{\rm r} - U^*_0$, which allows us to express the velocity of the reflected SW2 as a function $D_{\rm r} = D_{\rm r}(D_{\rm tr}, U_{\rm tr}(D_{\rm tr}))$ in the following form:

$$D_{\rm r} = U_{\rm fin} - U^*_0 + [P_{\rm fin} - \rho^{**}_0 U^{**}_0 (U^{**}_0 - U_{\rm tr})]/[\rho_{\rm fin}(U_{\rm fin} - U^*_0 + U^{**}_0 - U_{\rm tr})]. \tag{6}$$

We study the last equation of system (3) for determining the velocity of the transient (refracted) SW3. Substituting $P_{\rm tr}$ and $P_{\rm r}$ from the equation of state and performing a number of simple transformations, we obtain an equation for $\rho_{\rm tr}$ and $\rho_{\rm r}$:

$$C\rho^2_{\rm tr}\xi^{**}_1\xi^{**}_2 + C\rho^2_{\rm r}\xi^*_1\xi^*_2 + \rho_{\rm tr}\rho^2_{\rm r}(C^*_{\rm e.f.})^2(\xi^*_1\xi^{**}_2 - \xi^{**}_1\xi^*_2) - \rho^2_{\rm tr}\rho_{\rm r}(C^{**}_{\rm e.f.})^2(\xi^*_1\xi^{**}_2 - \xi^{**}_1\xi^*_2) - C\rho_{\rm tr}\rho_{\rm r}(\xi^*_1\xi^{**}_2 + \xi^{**}_1\xi^*_2) = 0. \tag{7}$$

Equation (7) is a function of one unknown — velocity of the transient SW3 $D_{\rm tr}$, since $\rho_{\rm tr}$ and $\rho_{\rm r}$ are expressed through the parameters $U_{\rm tr}$, $U_{\rm r}$, $U^{\rm r}_{\rm fin}$, and U^{**}_0, which, in turn, are functions of $D_{\rm tr}$. The roots of Eq. (7) were found graphically. Figure 3 shows the behavior of the roots of Eq. (7) as a function of the velocity of the refracted SW3 for different initial volume concentrations of the light component ahead of the CD boundary m^*_{10} for $D = -2.5$ and $m^{**}_{10} = 0.5$. It follows from Fig. 3 that, for all $m^*_{10} \in (0, 1)$, there are three constant roots corresponding to the state at rest ($D_{\rm tr} = 0$) and equilibrium velocity of sound ($D_{\rm tr} = \pm C^{**}_{\rm e}$). Two solutions (curves 1 and 2) exist for all values of $m^*_{10} = 1$–0. They decrease as the velocity of the transient wave changes from negative to positive values. The behavior of the remaining roots in Fig. 3 is shown by curve 3 (curve with a closed section). The vertical dashed line shows the velocity of the incident SW. The existence of a point m_* should also be noted; this point separates the region of unstable flow (located below the point m_*), where the condition of the Zemplén theorem is not satisfied. This value was determined in [3] by constructing a chart of solutions. Based on the above considerations, we formulate the following statement.

Statement 1. *Depending on the value of m^*_{10}, the region of existence of the solution of the problem of incidence of an* SW *onto the* CD *can be divided into three regions* (see Fig. 3).

In region I, *the volume concentrations of the first phase ahead of the* CD *are $m^*_{10} \in (0, m_*)$, which corresponds to the case of a rarefaction wave incident onto the* CD *boundary. For $m^*_{10} = m_*$, there exist three roots: $D_{\rm tr} = 0$ and $D_{\rm tr} = \pm C^{**}_{\rm e}$* (see Fig. 3). *It should be noted that the value of m_* depends only on the velocity of the wave interacting with the* CD.

TABLE 1

Equilibrium Parameters of the Mixture ($D = -1.5$)

m_{10}^*	$D_{\rm tr}$	$D_{\rm r}$	$u_{\rm CD}$	$u_{\rm fin}$	$P_{\rm CD}$	$P_{\rm fin}$	$\rho_{\rm fin}$	$\rho_{\rm tr}$	$\rho_{\rm r}$
				$m_{10}^{**} = 0.1$					
0.2	−1.879	1.512	−0.065	−0.075	0.303	0.263	2.443	2.574	2.459
0.3	−2.120	1.460	−0.167	−0.213	0.880	0.688	2.511	2.698	2.582
0.4	−2.282	1.351	−0.247	−0.338	1.401	1.008	2.568	2.787	2.714
0.5	−2.395	1.210	−0.310	−0.452	1.848	1.238	2.611	2.855	2.853
0.6	−2.475	1.048	−0.359	−0.555	2.209	1.382	2.635	2.907	3.002
0.7	−2.527	0.877	−0.394	−0.648	2.472	1.4524	2.631	2.949	3.158
0.8	−2.555	0.703	−0.412	−0.728	2.618	1.4520	2.584	2.963	3.314
0.9	−2.557	0.541	−0.414	−0.792	2.628	1.384	2.468	2.965	3.446
				$m_{10}^{**} = 0.5$					
0.2	−1.116	RW	−0.094	−0.075	0.192	0.263	2.443	1.994	2.413
0.3	−1.270	RW	−0.244	−0.213	0.566	0.688	2.511	2.259	2.462
0.4	−1.397	RW	−0.360	−0.338	0.919	1.008	2.568	2.459	2.531
0.5	*−1.500*	—	*−0.452*	*−0.452*	*1.238*	*1.238*	*2.611*	*2.611*	*2.611*
0.6	−1.581	0.884	−0.522	−0.555	1.507	1.382	2.635	2.725	2.697
0.7	−1.641	0.716	−0.574	−0.648	1.718	1.4524	2.631	2.806	2.782
0.8	−1.678	0.552	−0.606	−0.728	1.855	1.4520	2.584	2.856	2.856
0.9	−1.691	0.404	−0.616	−0.792	1.902	1.384	2.468	2.872	2.893

In region II*, the volume concentration of the first phase is* $m_{10}^* \in (m_*, m_{10}^{**})$. *Here, the incident SW1 propagates over a denser mixture and enters a less dense mixture at the CD boundary* (see Fig. 1b). *For* $m_{10}^* = m_{10}^{**}$, *there exist four roots:* $D_{\rm tr} = 0$, $D_{\rm tr} = \pm C_{\rm e}^{**}$, *and* $D_{\rm tr} = D$. *The latter root corresponds to the case, where there is no discontinuity in the volume concentration on the CD, and the incident SW1 passes through the CD without changes.*

Region III *corresponds to the values* $m_{10}^* \in (m_{10}^{**}, 1)$. *In this case, the incident SW1 propagates over a mixture with a lower density than behind the CD* (see Fig. 1a).

As the velocity of the incident SW decreases, region I increases: we have $m_* = 0.149$ for $D = -1.5$, $m_* = 0.058$ for $D = -2$, $m_* = 0.020$ for $D = -2.5$, and $m_* = 0$ for $D = -3$; region I is absent for $D < -3$. The width of regions II and III depends only on the volume concentration of the light (heavy) component behind the CD.

Among the solutions obtained, we chose solutions that satisfied the following conditions: first, SW3 should be stable, hence, its velocity should be greater than the equilibrium velocity of sound, i.e., $D_{\rm tr} \notin [-C_{\rm e}^{**}, C_{\rm e}^{**}]$; second, the direction of the transient SW3 should coincide with the direction of the incident SW1, i.e., $\operatorname{sign}(D_{\rm tr}) = \operatorname{sign}(D)$. Thus, among the set of solutions of the problem posed, only the solutions in the region of supersonic flows in terms of the equilibrium velocity of sound satisfy physical requirements. For the case presented in Fig. 3, this region can contain two to five roots of Eq. (7), depending on the value of m_{10}^*.

The results of numerical calculations in an unsteady approximation showed that only one root is obtained in practice, which is further used as the solution (curve 2 in Fig. 3). Using the solution for $D_{\rm tr}$, we determine the remaining equilibrium flow parameters.

Tables 1–3 contain the velocities of the transient and reflected SW, velocity of the CD, velocities of the mixture behind the incident SW1, pressure on the CD and in the final equilibrium state behind the incident SW1, and densities behind the incident, transient, and reflected shock waves for various differences in volume concentrations at the CD boundary and velocities of the incident shock wave. An analysis of the data in Tables 1–3 allows us to make the following conclusions. The velocity of the transient SW3 $D_{\rm tr}$ increases with increasing velocity of the incident SW1 and initial volume concentration of solid particles ahead of the CD boundary and decreases with increasing initial volume concentration of the light component. As it could be expected, the velocity of the transient SW3 is smaller than the velocity of the incident shock wave if the incident SW1 interacts with a less dense medium ($m_{10}^* < m_{10}^{**}$). The velocity of the reflected SW2 decreases with decreasing initial volume concentration of particles in the mixture; the degree of deceleration increases with increasing velocity of the incident SW1. It should be noted that, if a weaker SW ($D = -1.5$) is incident onto the CD boundary, the reflected SW2 always moves in the opposite direction. As the velocity of the incident SW1 increases ($D = -2.5$ and -3.3), in the case of high volume concentrations of the light component ahead of the CD boundary, the reflected SW2 is entrained by the flow

TABLE 2

Equilibrium Parameters of the Mixture ($D = -2.5$)

m^*_{10}	D_{tr}	D_{r}	u_{CD}	u_{fin}	P_{CD}	P_{fin}	ρ_{fin}	ρ_{tr}	ρ_{r}
				$m^{**}_{10} = 0.1$					
0.1	*−2.500*	—	*−0.375*	*−0.375*	*2.332*	*2.332*	*2.924*	*2.924*	*2.924*
0.2	−2.831	1.995	−0.631	−0.685	4.438	3.974	3.196	3.197	3.262
0.3	−3.017	1.707	−0.810	−0.928	6.072	5.002	3.428	3.397	3.589
0.4	−3.146	1.430	−0.949	−1.137	7.428	5.659	3.651	3.559	3.940
0.5	−3.246	1.151	−1.063	−1.325	8.573	6.048	3.885	3.695	4.345
0.6	−3.325	0.859	−1.157	−1.499	9.563	6.222	4.147	3.812	4.849
0.7	−3.389	0.542	−1.236	−1.662	10.408	6.213	4.462	3.911	5.533
0.8	−3.440	0.188	−1.299	−1.817	11.101	6.042	4.868	3.993	6.565
0.9	−3.475	−0.213	−1.344	−1.963	11.606	5.719	5.428	4.052	8.402
				$m^{**}_{10} = 0.5$					
0.1	−1.554	RW	−0.499	−0.375	1.414	2.332	2.924	2.687	2.789
0.2	−1.929	RW	−0.820	−0.685	2.887	3.974	3.196	3.176	3.032
0.3	−2.176	RW	−1.035	−0.928	4.108	5.002	3.428	3.480	3.286
0.4	−2.358	RW	−1.196	−1.137	5.148	5.659	3.651	3.705	3.563
0.5	*−2.500*	—	*−1.325*	*−1.325*	*6.048*	*6.048*	*3.885*	*3.885*	*3.885*
0.6	−2.614	0.647	−1.431	−1.499	6.828	6.222	4.147	4.032	4.283
0.7	−2.707	0.325	−1.518	−1.662	7.497	6.213	4.462	4.155	4.813
0.8	−2.779	−0.029	−1.587	−1.817	8.047	6.042	4.868	4.253	5.589
0.9	−2.831	−0.423	−1.636	−1.963	8.454	5.719	5.428	4.324	6.892

TABLE 3

Equilibrium Parameters of the Mixture ($D = -3.3$)

m^*_{10}	D_{tr}	D_{r}	u_{CD}	u_{fin}	P_{CD}	P_{fin}	ρ_{fin}	ρ_{tr}	ρ_{r}
				$m^{**}_{10} = 0.1$					
0.1	*−3.300*	—	*−1.128*	*−1.128*	*9.247*	*9.247*	*3.775*	*3.775*	*3.775*
0.2	−3.494	1.433	−1.367	−1.439	11.872	11.020	4.115	4.083	4.221
0.3	−3.626	1.137	−1.538	−1.687	13.856	11.996	4.409	4.314	4.655
0.4	−3.729	0.859	−1.672	−1.903	15.495	12.497	4.701	4.505	5.129
0.5	−3.814	0.580	−1.785	−2.101	16.917	12.654	5.023	4.670	5.696
0.6	−3.888	0.285	−1.883	−2.289	18.192	12.537	5.416	4.818	6.430
0.7	−3.954	−0.044	−1.971	−2.470	19.367	12.183	5.940	4.954	7.478
0.8	−4.015	−0.429	−2.052	−2.647	20.480	11.618	6.721	5.083	9.182
0.9	−4.073	−0.904	−2.130	−2.823	21.557	10.852	8.053	5.208	12.605
				$m^{**}_{10} = 0.5$					
0.1	−2.569	RW	−1.389	−1.128	6.511	9.247	3.775	3.973	3.450
0.2	−2.851	RW	−1.655	−1.439	8.611	11.020	4.115	4.352	3.812
0.3	−3.040	RW	−1.840	−1.687	10.207	11.996	4.409	4.623	4.168
0.4	−3.184	RW	−1.983	−1.903	11.522	12.497	4.701	4.840	4.559
0.5	*−3.300*	—	*−2.101*	*−2.101*	*12.654*	*12.654*	*5.023*	*5.023*	*5.023*
0.6	−3.398	0.089	−2.202	−2.289	13.654	12.537	5.416	5.183	5.621
0.7	−3.483	−0.248	−2.222	−2.470	14.556	12.183	5.940	5.326	6.463
0.8	−3.559	−0.638	−2.368	−2.647	15.380	11.618	6.721	5.456	7.804
0.9	−3.626	−1.114	−2.439	−2.823	16.136	10.852	8.053	5.574	10.386

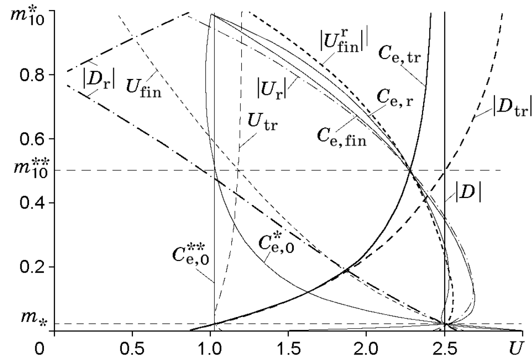

Fig. 4. Characteristic velocities of the mixture.

formed behind the incident SW1. Thus, with respect to a motionless observer, the reflected SW2 can move in the same direction as the transient SW3 but with a lower velocity (see Tables 1–3). The velocity of the CD boundary increases both with increasing velocity of the incident SW1 and with increasing initial volume concentrations of the light component ahead of and behind the CD boundary. The pressure on the CD increases with increasing m_{10}^*, m_{10}^{**}, and D; if $m_{10}^* > m_{10}^{**}$, then P_CD is higher than the pressure established behind the incident SW1; otherwise, $P_\mathrm{CD} < P_\mathrm{fin}$, and the pressure in the reflected wave decreases (a flow with a rarefaction wave is formed). The behavior of the density of the mixture is similar, i.e., for $m_{10}^* > m_{10}^{**}$, the density behind the refracted and reflected shock waves is greater than the density behind the incident SW1; for $m_{10}^* < m_{10}^{**}$, ρ_fin is always greater than ρ_r, and ρ_tr may be either greater or smaller than ρ_fin, depending on the velocity of the incident SW1 and the value of m_{10}^*.

Conditions of Flow Stability. Figure 4 shows the characteristic velocities of the mixture in the case of incidence of SW1 propagating with a velocity $D = -2.5$ over the mixture versus the volume concentration. The volume concentrations of the components behind the CD are identical ($m_{10}^{**} = m_{20}^{**} = 0.5$). The characteristic velocities are the velocity of the incident SW1 (solid vertical curve $|D|$), velocity of the refracted SW3 (dashed curve $|D_\mathrm{tr}|$), velocity of the reflected SW2 (dot-and-dashed curve D_r), mass velocity of the mixture behind the incident SW1 in the SW-fixed coordinate system (thin dotted curve $U_\mathrm{fin} = u_\mathrm{fin} - D$), mass velocity of the mixture behind the refracted shock wave in the relative coordinate system (thin dashed curve U_tr), relative equilibrium velocity ahead of the reflected SW (dotted curve $|U_\mathrm{fin}^\mathrm{r}|$), relative equilibrium velocity behind the reflected SW (thin dot-and-dashed curve $|U_\mathrm{r}|$), and equilibrium velocities of sound (thin solid curves $C_\mathrm{e,0}^*$, $C_\mathrm{e,0}^{**}$, $C_\mathrm{e,fin}$, $C_\mathrm{e,r}$, and $C_\mathrm{e,tr}$) in the initial states ahead of and behind the CD and in the states behind the incident, reflected, and refracted SW.

The equilibrium velocity of sound is calculated by the formula

$$C_\mathrm{e}^2 = \frac{\xi_1}{m_1}\,\frac{m_1 C - \rho\xi_1}{m_1^2 C - \rho\xi_1}, \qquad m_1 = m_1^e(\rho),$$

where $C = 1 - a^2\bar{\rho}$; the values of $C_\mathrm{e,0}^*$, $C_\mathrm{e,0}^{**}$, $C_\mathrm{e,fin}$, $C_\mathrm{e,tr}$, and $C_\mathrm{e,r}$ are determined by substituting equilibrium parameters of the mixture for the corresponding equilibrium states ahead of and behind the CD and behind the incident, transient, and reflected shock waves.

According to the Zemplén theorem, two conditions should be satisfied in the case of a steady flow: 1) the relative velocity of the incident SW is greater than the equilibrium velocity of sound in the mixture ahead of the SW front; 2) the relative velocity of the flow behind the front of the incident SW is smaller than the equilibrium velocity of sound in the final state. Thus, the incident SW1 is stable for $|D| > C_\mathrm{e,0}^*$ and $|u_\mathrm{fin} - D| < C_\mathrm{e,fin}$, the transient SW3 is stable for $|D_\mathrm{tr}| > C_\mathrm{e,0}^{**}$ and $|u_\mathrm{tr} - D_\mathrm{tr}| < C_\mathrm{e,tr}$, and the reflected SW2 is stable for $|u_\mathrm{fin} - D_\mathrm{r}| > C_\mathrm{e,fin}$ and $|u_\mathrm{r} - D_\mathrm{r}| < C_\mathrm{e,r}$. It follows from Fig. 4 that the incident SW1 propagates steadily over the mixture for $m_{10}^* \in (m_*, 1)$, the transient SW3 is stable on the same interval as the incident SW1, and the reflected SW2 is stable on the interval $m_{10}^* \in (m_{10}^{**}, 1)$. For the remaining values of m_{10}^*, the condition of the Zemplén theorem is not satisfied; therefore, the flow is unstable. From the above analysis, there follows

Statement 2. *For $m_{10}^* \in (m_{10}^{**}, 1)$, the incident, transient, and reflected shock waves are stable* (see Fig. 1a).

For $m_{10}^* \in (m_*, m_{10}^{**})$, the incident and transient shock waves are stable, and the RW *is reflected from the* CD (see Fig. 1b).

For $m_{10}^* \in (0, m_*)$, the wave pattern with incident, transient, and reflected rarefaction waves is formed.

TABLE 4

Equilibrium Parameters of the Mixture behind the Incident and Reflected SW for $D = -2.5$ (rigid wall)

m^*_{10}	$u_{\rm fin}$	$P_{\rm fin}$	$D_{\rm r}$	$P_{\rm r}$	$k_{\rm CD}/k_W$
0.1	−0.375	2.332	2.622	5.622	0
0.2	−0.685	3.974	2.446	10.828	0.0677
0.3	−0.928	5.002	2.271	15.186	0.105
0.4	−1.138	5.659	2.095	19.084	0.132
0.5	−1.326	6.048	1.909	22.700	0.152
0.6	−1.499	6.222	1.704	26.135	0.168
0.7	−1.662	6.213	1.470	29.449	0.181
0.8	−1.817	6.042	1.191	32.655	0.190
0.9	−1.964	5.719	0.846	35.662	0.197

We consider the limiting case, where there is no difference in volume concentrations at the CD boundary, i.e., $m^*_{10} = m^{**}_{10}$. This case corresponds to SW propagation over a two-component mixture. The velocity of the transient SW is equal to the velocity of the incident SW ($D_{\rm tr} = D$), and the velocity of the reflected SW is determined as $D_{\rm r} = u_{\rm r} - U_{\rm r}$. The CD velocity is equal to the mass velocity of the mixture behind the incident SW ($u_{\rm CD} = u_{\rm fin}$). In this case, the equilibrium parameters of the mixture are $P_{\rm CD} = P_{\rm fin}$, $\rho_{\rm r} = \rho_{\rm tr} = \rho_{\rm fin}$, $U^{\rm r}_{\rm fin} = U_{\rm r} = -C_{\rm e,fin}$, and $U_{\rm tr} = U_{\rm fin}$. The values of parameters corresponding to this case are italicized in Tables 1–3.

We compare the results obtained with the data for the limiting case, where an absolutely impermeable body is located behind the CD boundary, i.e., the incident SW1 interacts with a rigid wall. The problem of SW interaction with a rigid wall in the equilibrium approximation was studied in [5].

Table 4 shows the equilibrium parameters of the mixture behind the incident SW1 and the parameters established after SW interaction with a rigid wall for $D = -2.5$ and various m^*_{10}. It follows from Table 4 that the velocity $D_{\rm r}$ of the shock wave reflected from the rigid wall is greater than the velocity of the shock wave reflected from the CD boundary (see Table 2). Note, in the case examined, it is not possible to change the direction of motion of SW2 reflected from the rigid boundary. Table 4 contains also the attenuation coefficients for SW2 reflected from the CD ($k_{\rm CD}/k_W$), which is the ratio of the pressure increment due to interaction of the incident SW1 and CD behind which we have $m^{**}_{10} = 0.1$ ($P_{\rm CD} - P_{\rm fin}$) to the pressure increment due to SW interaction with the rigid wall ($P_{\rm r} - P_{\rm fin}$). The coefficient $k_{\rm CD}/k_W$ increases from zero in the case of the absence of the difference in volume concentrations on the CD ($\Delta m = 0$) to 0.197 for $\Delta m = 0.8$.

Asymptotic Solutions. We study the limiting solutions of the problem posed, when the volume concentrations of the components of the mixture tend to zero and unity. Three variants are possible: 1) the incident SW1 propagates over a two-component mixture and interacts with a homogeneous material; 2) SW1 moves in a pure material and interacts with a two-component mixture; 3) SW1 propagates over a pure one-component material and interacts with a homogeneous material.

We consider the first case, where the incident SW1 propagates over a two-component mixture and interacts with the CD with a homogeneous material behind. The equation of state for the homogeneous material behind the CD boundary is $P^{**} = (a^{**})^2(\rho - \rho^{**}_0)$, where a^{**} and ρ^{**}_0 are the dimensionless velocity of sound and density in the homogeneous material. This equation of state is used in solving system (3). The expressions for the parameters of the mixture are written as follows:

— $U_{\rm tr} = -(a^{**})^2/D_{\rm tr}$ for velocity of the transient SW;

— $U_{\rm r} = (a^{**})^2/U^{**}_0 + D_{\rm tr} - D_{\rm r}$ for velocity of the mixture behind the reflected SW;

— $D_{\rm r} = U_{\rm fin} - U^*_0 + \{P_{\rm fin} - \rho^{**}_0[D^2_{\rm tr} - (a^{**})^2]\}/[\rho_{\rm fin}(U_{\rm fin} - U^*_0 - D_{\rm tr} - U_{\rm tr})]$ for velocity of the reflected SW.

The condition of equal pressures on the CD yields the following equation for determining the velocity of the transient SW $D_{\rm tr}$:

$$[1 + \rho^{**}_0(D^2_{\rm tr} - (a^{**})^2)]^2 - [1 + \rho^{**}_0(D^2_{\rm tr} - (a^{**})^2)][C + \rho_{\rm r}(C^*_{\rm e.f.})^2] + C\rho_{\rm r}\xi^*_1 = 0.$$

Here $\rho_{\rm r} = \rho_{\rm fin}(U_{\rm fin} - U^*_0 - D_{\rm r})/(D_{\rm tr} - D_{\rm r} + U_{\rm tr})$. Figure 5 shows the behavior of the roots of this equation (curves 1–5) versus the velocity of the transient SW $D_{\rm tr}$ for the velocity of the incident SW $D = -2.5$. The velocity of sound and the true density for the pure material are $a^{**} = 3000$ m/sec and $\rho^{**} = 1825$ kg/m^3 (in the dimensionless form, $a^{**} = 2$ and $\rho^{**} = 1.825$). There are solutions corresponding to the state at rest ($D_{\rm tr} = 0$) and the velocity of sound in the pure material behind the CD ($D_{\rm tr} = \pm a^{**}$); two of them are not plotted in Fig. 5. The

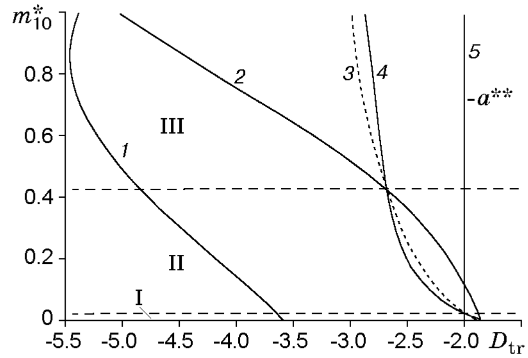

Fig. 5. Velocities of the SW passing from the mixture to the homogeneous material.

TABLE 5

Velocities of the SW Passing from the Pure Material into the Mixture

$D_{\rm tr}$	$m^{**}_{10} = 0.3$		$m^{**}_{10} = 0.5$		$m^{**}_{10} = 0.7$	
	Equation (8)	Equation (7)	Equation (8)	Equation (7)	Equation (8)	Equation (7)
$D^{\rm I}_{\rm tr}$	−6.345	−6.124 −6.342	−5.408	−5.006 −5.404	−4.340	−3.771 −4.335
$D^{\rm II}_{\rm tr}$	−3.129	−3.107 −3.129	−2.857	−2.831 −2.857	−2.660	−2.630 −2.660
$D^{\rm III}_{\rm tr}$	−2.709	−2.754 −2.710	−2.552	−2.593 −2.553	−2.462	−2.502 −2.463
$D^{\rm IV}_{\rm tr}$	−1.187	−1.187	−1.026	−1.026	−0.969	−0.969
$D^{\rm V}_{\rm tr}$	1.187	1.187	1.026	1.026	0.969	0.969
$D^{\rm VI}_{\rm tr}$	3.902	4.264 3.906	6.411	8.848 6.428	9.972	—

Note. In columns 3, 5, and 7, the first and second values were obtained for $m^*_{10} = 0.9$ and 0.999, respectively.

horizontal dashed lines in Fig. 5 show the boundaries of regions I–III (see Statement 1). Note that the position of the upper boundary corresponds to the condition of equal densities in the pure material and the mixture ($\rho^{**} = \rho^*_0$). The root obtained in numerical calculations of the unsteady problem is shown by curve 3.

We consider now the second limiting case, where the shock wave propagates over a pure one-component substance and interacts with a mixture of two condensed materials. We use the equation of state for the pure material in the form $P^* = (a^*)^2(\rho - \rho^*_0)$, where a^* and ρ^*_0 are dimensionless velocity of sound and density in the homogeneous material. In this case, the flow velocity behind the incident SW is determined as $U_{\rm fin} = (a^*)^2/U^*_0$, and the expressions for density and pressure have the form $\rho_{\rm fin} = (U^*_0)^2\rho^*_0/(a^*)^2$ and $P_{\rm fin} = \rho^*_0((U^*_0)^2 - (a^*)^2)$, respectively. The velocity of the mixture behind the transient SW3 is determined from Eq. (4), that behind the reflected SW2 from Eq. (5), and the velocity of the reflected SW2 from Eq. (6). The fourth equation of system (3) used to determine the velocity of the transient SW $D_{\rm tr}$ is written as

$$[1 + (a^*)^2(\rho_{\rm r} - \rho^*_0)]^2 U_{\rm tr} - [1 + (a^*)^2(\rho_{\rm r} - \rho^*_0)][CU_{\rm tr} + \rho^{**}_0 U^{**}_0 (C^{**}_{\rm e.f.})^2] + C\rho^{**}_0 U^{**}_0 \xi^{**}_1 = 0, \qquad (8)$$

where $\rho_{\rm r} = (\rho^*_0 U^*_0/(a^*)^2)((a^*)^2 - D_{\rm r} U^*_0 - (U^*_0)^2)/(U_{\rm tr} + D_{\rm tr} - D_{\rm r})$; $U_{\rm tr}$ is determined from Eq. (4).

The solutions of Eq. (8) corresponding to the case of a pure liquid present ahead of the CD ($a^* = 1$ and $\rho^{**}_0 = 1$) can be obtained from Eq. (7) considered above, as $m^*_{10} \to 1$. The results of the numerical solution of Eq. (7) (in determining the roots of the equation for $D_{\rm tr}$, $D_{\rm r}$, $U_{\rm r}$, $U_{\rm tr}$, $u_{\rm CD}$, $u_{\rm fin}$, $P_{\rm CD}$, $P_{\rm fin}$, $\rho_{\rm fin}$, $\rho_{\rm tr}$, and $\rho_{\rm r}$) obtained for $m^*_{10} = 1 - \varepsilon$ differ insignificantly from the results of the solution of Eq. (8). In particular, for $D = -2.5$ and $m^{**}_{10} = 0.3$, 0.5, and 0.7, we found six roots (I–VI) of Eq. (8) within the range of velocities of the transient SW from −10 to 10 (Table 5). For each value of m^{**}_{10}, Table 5 also gives solutions obtained from Eq. (7) for $m^*_{10} = 0.9$ and 0.999. It follows from Table 5 that the roots $D^{\rm IV}_{\rm tr}$ and $D^{\rm V}_{\rm tr}$ correspond to the values $-C^{**}_{\rm e}$ and $C^{**}_{\rm e}$ (velocity of sound in the mixture behind the CD). With increasing m^*_{10}, the solution obtained from Eq. (7) approaches the exact solution obtained from Eq. (8). In numerical calculations of the unsteady problem, we obtain the root $D^{\rm II}_{\rm tr}$. Variation of the roots $D_{\rm tr}$ within the interval $m^*_{10} = 0.9$–0.999 for the general solution of (7) with $m^{**}_{10} = 0.5$ is plotted in Fig. 3. Note, in the example considered, the pure material has always a lower density than the mixture; therefore, the wave pattern shown in Fig. 1a is always observed.

We study the third case, where we have two pure materials separated by the CD. For definiteness, we assume that $m^{**}_{10} \to 0$ and $m^*_{10} \to 1$. Fur pure materials, we use the equations of state of the form $P_i = a_i^2(\rho_i - \rho_{i0})$, where the subscripts $i = 1$ and 2 refer to materials on the right of the CD (liquid phase) and on the left of it (solid phase). The equations were normalized to the values of parameters of the liquid phase. An SW moves over material 1 with a velocity D, and a state with the following gas-dynamic parameters is established behind the SW front: $U_{\rm fin} = 1/U_{10}$, $\rho_{\rm fin} = U^2_{10}$, and $P_{\rm fin} = U^2_{10} - 1$. From the conditions on the transient SW, we determine the parameters $U_{\rm tr} = a^2/U_{20}$, $\rho_{\rm tr} = \bar\rho U^2_{20}/a^2$, and $P_{\rm tr} = \bar\rho(U^2_{20} - a^2)$. From the conditions on the reflected SW, we have $U_{\rm r} = 1/U^{\rm r}_{\rm fin}$, $\rho_{\rm r} = U^2_{10}(U^{\rm r}_{\rm fin})^2$, and $P_{\rm r} = U^2_{10}(U^{\rm r}_{\rm fin})^2 - 1$. Using the conditions on the CD after its interaction with the incident SW, we obtain two expressions: in the case of equal pressures on the CD, the velocity of the reflected SW $D_{\rm r} = D_{\rm r}(U_{20}(D_{\rm tr}))$ is written as $D_{\rm r} = U_{\rm fin} - U_{10} \mp \sqrt{\bar\rho(U^2_{20} - a^2) + 1}/U_{10}$, and in the case of equal mass velocities on the CD, the velocity of the transient SW $D_{\rm tr}$ is determined from the relation $U_{\rm tr} + D_{\rm tr} = U_{\rm r} + D_{\rm r}$. Substituting the previously obtained expressions for $U_{\rm tr}$, $U_{\rm r}$, and $D_{\rm r}$, which depend on $D_{\rm tr}$ only, we obtain a transcendental equation, which has four real roots for an SW propagating with a velocity $D = -2.5$: -7.671, -3.715, -3.000, and 3.000. The absolute value of two last solutions corresponds to the velocity of sound in the solid material behind the CD. Numerical experiments showed that the second root is actually obtained. Thus, the following parameters are established when the SW passes from the liquid to the solid material: $D_{\rm tr} = -3.715$, $D_{\rm r} = -0.618$, $U_{\rm fin} = 0.400$, $U_{\rm tr} = 2.422$, $U_{\rm r} = -0.675$, $\rho_{\rm fin} = 6.250$, $\rho_{\rm tr} = 4.064$, $\rho_{\rm r} = 13.727$, $P_{\rm fin} = 5.250$, $P_{\rm CD} = 12.727$, and $u_{\rm CD} = -1.292$. These results coincide with those obtained by solving Eq. (7) for $m^{**}_{10} = 0.0001$ and $m^*_{10} = 0.9999$.

Conclusions. Within the framework of the mathematical model of the equilibrium approximation of mechanics of heterogeneous media for a mixture of two materials, the problem of interaction of an SW and a CD separating two two-component mixtures with different volume concentrations of heavy particles is considered. In particular,

— an asymptotic solution (in terms of relaxation times of velocities and pressures tending to zero) is obtained, which allows one to determine the wave pattern and flow parameters of the mixture, which are established after SW interaction with the CD;

— a chart of flows of the mixture is constructed, which includes three regions with different flow types;

— the possibility of using the results obtained for predicting the behavior of shock waves passing from a homogeneous material into a mixture or from a mixture into a homogeneous material is demonstrated; i.e., an asymptotic flow pattern is obtained for the volume concentration of one component tending to zero or unity.

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